Jan 2, 2019
Math 220 Sec 204

My name: Krishanu Sankar

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absolutely fine!

Email: ksankar@math.ubc.ca.

Course Website: - math.ubc.ca/~ollivier/Math220-2019.html

Textbook: Free to download. Follow instructions on website
Homework: The first one is due Fri. Jan 11th. (Details to come)

Office Hours: Please fill the email survey to determine my office hours!

My course website: math.ubc.ca/~ksankar/Math220-2019.html. For scanned lecture notes and worksheets

Lecture policies:
- Bring paper and (pen or pencil) as we will have in-class exercises
- If you must use a laptop or phone, please sit near the back. Screens are distracting!
- Food/drink is okay, just keep it quiet and neat.

Mathematical Questions

Q: Calculate the first digit of 7^{12}

Q: Calculate $\int_3^{10} x^3 dx$

Q: Can 63 be written as the sum of two squares of integers?

Done by finite calculation.

Q: Is $\sqrt{2}$ the ratio of some two whole numbers?

Q: Are there infinitely many prime numbers?

These questions cannot be answered just using calculation!

Proof: A 100% airtight argument that a statement is true (Hard!)

Example:

Proposition: Let n be any odd integer. Then n^2-1 is a multiple of 4.

(Note: odd^2 - 1 is even.)

Example: $n=1 \rightsquigarrow 1^2-1 = 0$ is a mult of 4
$n=3 \rightsquigarrow 3^2-1 = 8$ is ——
$n=5 \rightsquigarrow 5^2-1 = 24$ ——
$n=7 \rightsquigarrow 7^2-1 = 48$ ——
⋮

Not a proof

Proof: If n is an odd integer, then n can be written as $n = 2k+1$ for some integer k. (Words)

Then (Calculation)

$$\begin{aligned} n^2-1 &= (2k+1)^2 - 1 \\ &= 4k^2+4k+1-1 \\ &= 4k^2+4k \\ &= 4(k^2+k) \end{aligned}$$

Since k^2+k is an integer, it follows n^2-1 is a multiple of 4. ■ ← done. (Words)

Sets (Ch 1)

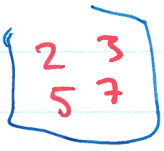
A set is a collection of objects.

Note: Define ours sets precisely! Can it be misinterpreted?

Ex: $S = \{2, 3, 5, 7\}$

↑ denote sets using capital

↑ list out elements between { }

Intuition: $S =$ 

Note: Order doesn't matter.

$\{5, 3, 7, 2\}$ means the same thing.

Ex: S = the set of positive integers which are strictly smaller than 7. $= \{1, 2, 3, 4, 5, 6\}$

↑ Useful for large sets

Ex: S = the set of all positive integers. ← Infinite set!

Note: Repeats are disregarded.

$\{2, 2, 3, 5, 7, 7\} = \{2, 3, 5, 7\}$

I want to be able to ask:

"Is x in the set S?"

Notation: $x \in S$ means "x is an element of S"

$x \notin S$ "x is not an element of S"

Two standard sets: $\{1, 2, 3, 4, 5, \ldots\} = \mathbb{N}$ ← natural numbers

$\{\ldots, -2, -1, 0, 1, 2, \ldots\} = \mathbb{Z}$ ← integers

Set Builder Notation

The set of odd integers $= \{\ldots, -3, -1, 1, 3, \ldots\}$ ... can be ambiguous! (honestly, using)

$S = \{2n+1 \mid n \in \mathbb{Z}\}$

this is the object that's in S

this is the simpler set we are using to build S.

Q: The set of all perfect squares $= \{0, 1, 4, 9, 16, 25, \ldots\}$

both of these are correct!

$= \{n^2 \mid n \in \mathbb{Z}\}$ ← has repeats but this is ok

$= \{n^2 \mid n \in \mathbb{Z} \text{ and } n \geq 0\}$